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Special class on

A.P., Quiz and introduction to G.P.

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ARITHMETIC PROGRESSION AND INTRODUCTION TO GEOMETRIC PROGRESSION

Introduction

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A sequence of the form a, a+d, a+2d, a+3d, …, is an Arithmetic Progression where
"a" is the first term and
"d" is the common difference
For example, 1, 4, 7, 10, …, is an A.P with a = 1 and d = 3

General term or n^{th} term of an A.P

$$a_n = a + (n - 1) d$$

$$S_n = \frac{n}{2} (a + a_n)$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Q. Find the sum of $34 + 32 + 30 + . . . + 10$

Sol. Clearly $34, 32, 30, . . ., 10$ is an A.P.,

where $a = 34$, $d = 32 - 34 = -2$

and $a_n = 10$

$$a + (n - 1)d = 10$$

$$34 + (n - 1)(-2) = 10$$

$$34 - 2n + 2 = 10$$

$$36 - 10 = 2n$$

$$26 = 2n$$

$$n = 13$$

$$S_n = \frac{n}{2}(a + a_n)$$

$$S_{13} = \frac{13}{2}(34 + 10)$$

$$= \frac{13}{2} \times 44$$

$$= 13 \times 22$$

$$= 286$$

The common difference of the A.P., for which $a_n = 3n + 1$ is

(A) 2 (B) 3

(C) 4 (D) 5

Q. Find the sum : $-5 + (-8) + (-11) + ... + (-230)$

Sol. Clearly $-5, (-8), (-11), ..., (-230)$ is an A.P., where

$a = -5$, $d = -8 - (-5) = -3$ and

$$a_n = -230$$

$$a + (n-1)d = -230$$

$$-5 + (n-1)(-3) = -230$$

$$-5 - 3n + 3 = -230$$

$$-2 - 3n = -230$$

$$-2 + 230 = 3n$$

$$228 = 3n$$

$$76 = n$$

$$S_n = \frac{n}{2}(a + a_n)$$

$$S_{76} = \frac{76}{2}\{(-5) + (-230)\}$$

$$= 38(-5 - 230)$$

$$= 38(-235)$$

$$= -8930$$

The common difference of the A.P., for which $a_n = 5n - 1$ is

(A) 2 (B) 3

(C) 4 (D) 5

Q. The sum of first six terms of an A.P. is 42. The ratio of its 10^{th} term to its 30^{th} term is 1:3. Calculate the first and the thirteenth term of this A.P.

Sol. Given, $S_6 = 42$

$$\frac{6}{2}[\,2a + (6-1)d\,] = 42$$

$$3\,(2a + 5d) = 42$$

$$2a + 5d = 14 \qquad -\text{(i)}$$

Also, $a_{10} : a_{30} = 1 : 3$

$$\frac{a+9d}{a+29d} = \frac{1}{3}$$

$$3\,(a + 9d) = a + 29d$$

$$3a + 27d = a + 29d$$

$$2a = 2d$$

$$a = d$$

Substituting a = d in eq. (i)

$2a + 5a = 14$

$7a = 14$

$a = 2$

$d = 2$ (as a = d)

Therefore, as a = 2 and d = 2, the A.P formed is 2, 4, 6, 8, 10, . .

$a = 2$

$a_{13} = a + 12d = 2 + 12\ (2)$

$= 2 + 24$

$= 26$

Hence, the first term is 2 and 13th term is 26.

The sum of first n natural numbers is

(A) $n(n+1)$

(B) $n(n-1)$

(C) $\frac{n}{2}(n+1)$

(D) $\frac{n}{2}(n-1)$

Q. How many terms of the series 54, 51, 48, . . . be taken so that their sum is 513 ? Explain the double answer.

Sol. Clearly the sequence 54, 51, 48, . . . is an A.P., where a = 54, d = 51 – 54 = – 3

$$S_n = 513$$

$$\frac{n}{2}[\,2\,(54) + (n-1)\,(-3)\,] = 513$$

$$\frac{n}{2}[\,108 - 3n + 3\,] = 513$$

$$\frac{n}{2}[\,111 - 3n\,] = 513$$

$$n\,(\,111 - 3n) = 513 \times 2$$

$$111n - 3n^2 = 1026$$

$$0 = 3n^2 - 111n + 1026$$

$$0 = n^2 - 37n + 342$$

$0 = n^2 - (18n + 19n) + 342$

$0 = n^2 - 18n - 19n + 342$

$0 = n(n - 18) - 19(n - 18)$

$0 = (n - 18)(n - 19)$

$n = 18, \quad n = 19$

Now, $a_n = a + (n - 1)d$

$a_{19} = 54 + 18(-3)$

$a_{19} = 54 - 54 = 0$

Hence, sum of 18 or 19 terms = 513, because $a_{19} = 0$.

The common difference of the A.P., $\frac{1}{2b}$, $\frac{1-6b}{2b}$, $\frac{1-12b}{2b}$, . . . is

(A) 2b

(B) – 2b

(C) – 3b

(D) – 3

Q. Solve $1 + 4 + 7 + . . . + x = 287$

Sol. Clearly, 1, 4, 7, . . . , x is an A.P., where $a = 1$, $d = 4 - 1 = 3$ and

$$S_n = 287$$

$$\frac{n}{2}(1 + x) = 287 \qquad -\text{(i)}$$

Also, $S_n = 287$

Therefore, $\frac{n}{2}[2(1) + (n - 1)(3)] = 287$

$$n[2 + (n - 1)3] = 547$$

$$n(2 + 3n - 3) = 547$$

$$n(3n - 1) = 547$$

$$3n^2 - n = 547$$

$$3n^2 - n - 547 = 0$$

$$3n^2 - (42n - 41n) - 547 = 0$$

$$3n^2 - 42n + 41n - 547 = 0$$

287×2

574

$3n(n - 14) + 41(n - 14) = 0$

$(n - 14)(3n - 14) = 0$

$n = 14, \quad n = 14/3$ (rejected)

Therefore, $n = 14$

Substituting $n = 14$ in eq.(i)

$$\frac{14}{2}(1 + x) = 287$$

$$7(1 + x) = 287$$

$$1 + x = 41$$

$$x = 40$$

The common difference of an A.P., is 5. Then what is $a_{10} - a_7$

(A) 3

(B) 10

(C) 15

(D) 5

Q. The ratio of the sum of n terms of two A.P's is $(7n+1):(4n+27)$. Find the ratio of their 10th terms

Sol. Let a, A be the first terms and d, D be the common differences of the two given A.P's.

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$$

$$\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$$

Dividing the numerator and the denominator on the LHS by 2, we get

$$\frac{a+\frac{(n-1)}{2}d}{A+\frac{(n-1)}{2}D} = \frac{7n+1}{4n+27}$$

$a + 9d = a_{10}$

$$\frac{n-1}{2} = 9 \Rightarrow n-1 = 18$$

$$\Rightarrow n = 19$$

$$\frac{a+9d}{A+9D} = \frac{7\times 19+1}{4\times 19+27}$$

$$= \frac{133+1}{76+27}$$

$$= \frac{134}{103}$$

$$= 134 : 103$$

How many 3 digit natural numbers leave remainder 1 when divided by 4 ?

(A)250

(B) 255

(C) 225

(D) none of these

Q. The sum of $n, 2n, 3n$ terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$

Sol. $RHS = 3(S_2 - S_1)$

$$= 3\left[\frac{2n}{2}\{2a+(2n-1)d\} - \frac{n}{2}\{2a+(n-1)d\}\right]$$

$$= \frac{3n}{2}[2(2a+2nd-d) - 2a - nd + d]$$

$$= \frac{3n}{2}(4a+4nd-2d-2a-nd+d)$$

$$= \frac{3n}{2}(2a+3nd-d)$$

$$= \frac{3n}{2}[2a+(3n-1)d]$$

$$= S_3 = LHS$$

Hence $S_3 = 3(S_2 - S_1)$

The sum of first 200 natural numbers is

(A) 10100

(B) 25250

(C) 20100

(D) 21000

QUIZ

For the A.P. 1, 5, 9, 13, . . . , $a_{10} - a_5$ is?

(A) 20

(B) 3

(C) 30

(D) 5

If $x + 2$, $2x$, $2x + 3$ are three consecutive terms of an A.P., then $x = ?$

(A) 5

(B) 4

(C) 3

(D) 2

Which term of the A.P., 21, 18, 15, . . ., is 0 ?

(A) 11^{th} (B) 10^{th}

(C) 9^{th} (D) 8^{th}

Two A.P.s have the same common difference. The difference between their 100^{th} terms is 111222333. What is the difference between their millionth terms ?

(A)11122233300

(B) 1112223330

(C) 111222333000

(D) 111222333

9^{th} term of the sequence defined by $a_n = (-1)^{n-1} n^3$ is

(A)729

(B) – 729

(C) 27

(D) – 27

For an A.P., if, $a_n = 2n + 1$, what is the sum of first three terms ?

(A) $6n + 3$

(B) 15

(C) 12

(D) 21

Geometric Progression

A sequence of the form a, ar, ar^2, ar^3, . . . is a Geometric Progression(G.P.) where, a is the first term r is the common ratio

For example, 2, 4, 8, 16, . . . is a G.P., with a = 2 and r = 2

Similarly, 2, 10, 50, 250, . . . is a G.P., with a = 2 and r = 5

3, 9, 27, 81, . . . is not a G.P.

Yes / No

The missing term of the G.P., $\sqrt{2}$, $2\sqrt{2}$, $\square$, $8\sqrt{2}$ is

(A) $3\sqrt{2}$

(B) $\sqrt{18}$

(C) $\sqrt{32}$

(D) $\sqrt{64}$

n^{th} term of the G.P.,

$a, ar, ar^2, ar^3, \ldots$ is

$$a_n = ar^{n-1}$$