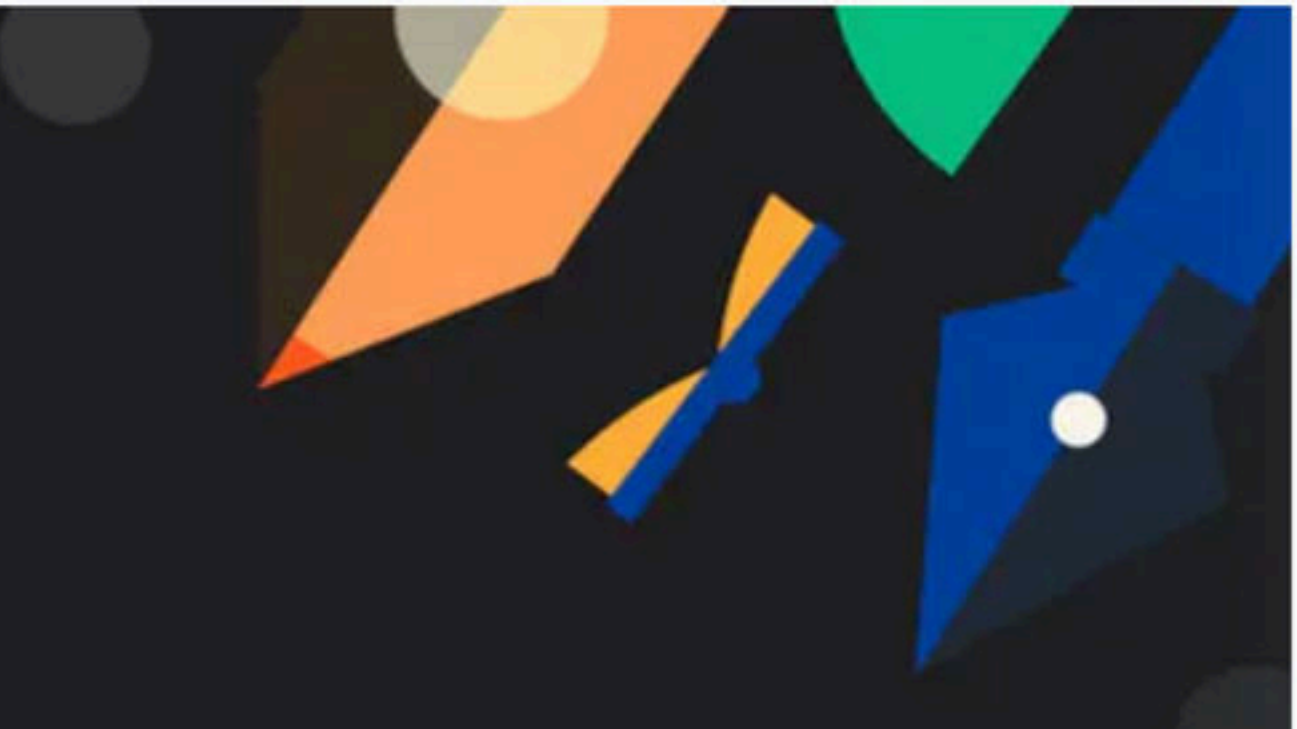




Geotechnical practice session-XV

Special class

Chetan Saxena • Nov 1, 2021

Factor of Safety w.r.t. Shear Strength (F_s)

- $F_s = \frac{C + \bar{\sigma} \tan \phi}{\tau}$ where, τ = Developed shear strength.
($C + \bar{\sigma} \tan \phi$) = Developed or mobilized shear stress

C = Effective cohesion

ϕ = Effective friction

$\bar{\sigma}$ = Effective normal stress

- $\tau = C_m + \bar{\sigma} \tan \phi_m$ where, C_m = Mobilized Cohesion

ϕ_m = Mobilized Friction Angle

$$C_m = \frac{C}{F_s}$$

and

$$\tan \phi_m = \frac{\tan \phi}{F_s}$$

ϕ_m = The strength tested, or developed, out of total.

Factor of Safety w.r.t. Cohesion (F_c)

$$F_c = \frac{H_c}{H} \text{ and } F_c = \frac{C}{C_m} \text{ where, } H_c = \text{Critical depth}$$

H = Actual depth

$$H_c = \frac{4C}{\gamma} \tan \left(45^\circ + \frac{\phi}{2} \right)$$

Stability Analysis of Infinite Slopes

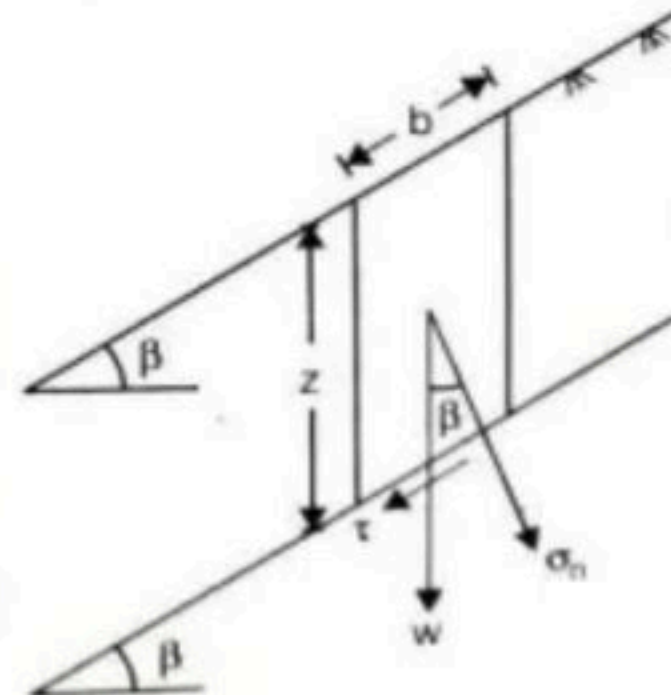
(i) Cohesionless dry soil/dry sand

Wt. of slope

$$W = \gamma Z \cos \beta$$

$$\tau = \frac{W \sin \beta}{(b \times 1)} \Rightarrow \tau = \gamma Z \sin \beta \cos \beta$$

$$\sigma_n = \frac{W \cos \beta}{(b \times 1)} \Rightarrow \sigma_n = \gamma Z \cos^2 \beta$$



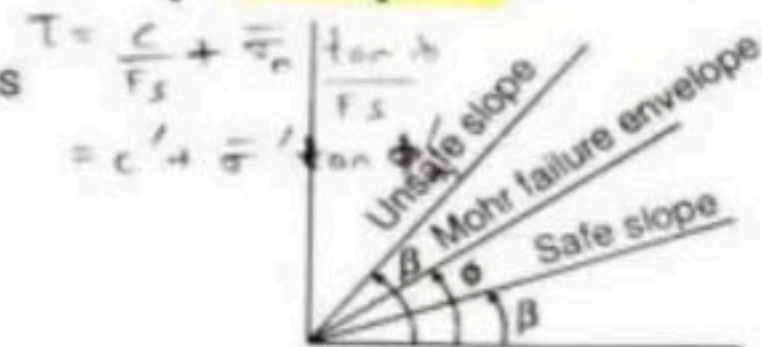
Infinite slope

finite slope

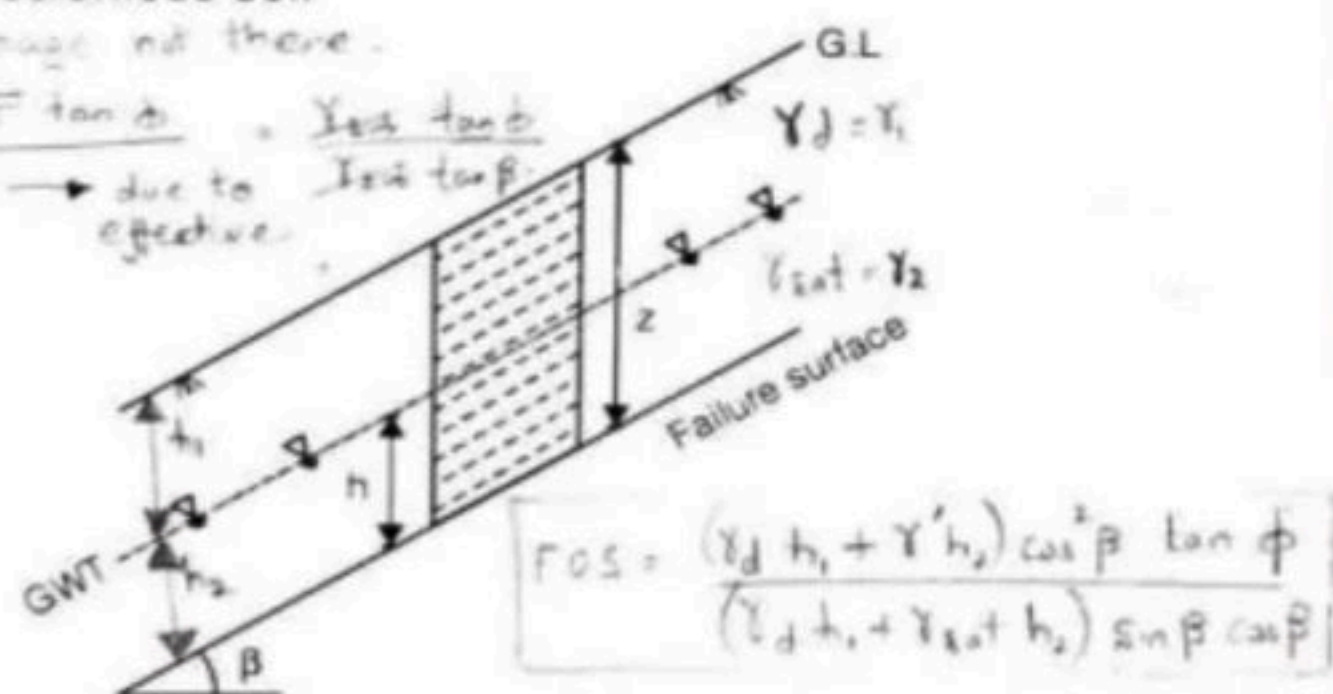
$$\tau < \tau_f$$

$$FOS = \frac{\tau_f}{\tau} > 1$$

$$F_s = \frac{\tan \phi}{\tan \beta}$$
$$r_s = \frac{S}{\tau} \pm \frac{C + \sigma_n \tan \phi}{\tau} \quad \text{--- (1)}$$
$$\beta < \phi$$

$$F_c > 1$$


- $\overline{I}_1$ sequence not there.

$$F_{\text{as}} = \frac{\sigma \tan \phi}{1} \rightarrow \text{due to effective}$$

$$F_s = \left[1 - \left(\frac{\gamma_w}{\gamma} \right) \left(\frac{h}{z} \right) \right] \frac{\tan \phi'}{\tan \beta}$$

γ —avg. total unit weight of soil above the slip surface upto ground level.

$$\gamma = \frac{\gamma_1 h_1 + \gamma_2 h_2}{h_1 + h_2}$$

$$F_s = \frac{\gamma'}{\gamma_{\text{Sat}}} \cdot \frac{\tan \phi}{\tan \beta} \quad F_s \approx \frac{1}{2} \cdot \frac{\tan \phi}{\tan \beta}$$

In formula ①

$$F_s = F_c = \frac{C}{yz \sin \beta \cdot \cos \beta} \quad F_c = \frac{H_c}{H}$$
At Critical Stage $F_c = 1$

$$F_c = \frac{H_c}{H} = \frac{C}{c_m}$$

$$\frac{C}{\gamma H_c} \left/ \frac{C_p F_c}{\gamma H_c} \right/ \frac{C_c}{\gamma F_c H} \left/ \frac{C_m}{\gamma H} \right/ S_n = \frac{C}{\gamma H_c} = \sin \beta \cdot \cos \beta = \frac{C}{\gamma F_c H} = \frac{C}{\gamma F_c z}$$

where S_n = Stability Number.

$$F_s = \frac{C}{\gamma H \sin \beta \cdot \cos \beta} + \frac{\tan \phi}{\tan \beta}$$

(vi) Taylor's stability no. (• F.O.S. with friction = 1)
Effective angle = actual incl angle

$$S_n = \frac{C}{\gamma H_c} = \sin \beta \cdot \cos \beta \quad (\text{for cohesive soil})$$

Max. theoretical value of stability no. = 0.5

Max. practical value is = 0.261

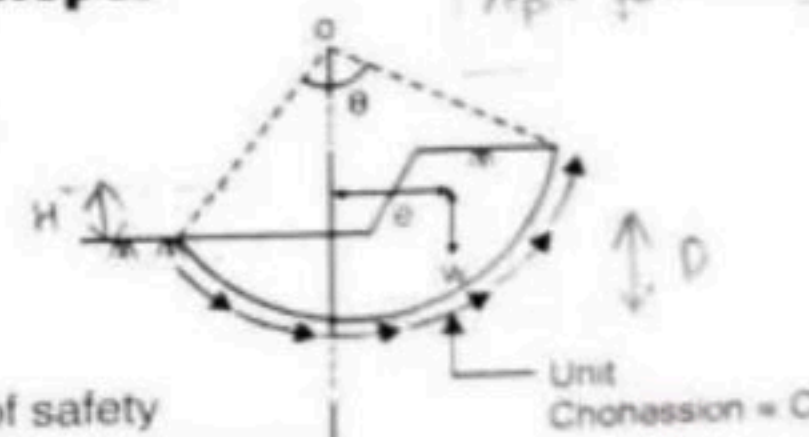
$$S_n = [\tan \beta - \tan \phi] \cos^2 \beta \quad (\text{for C-}\phi \text{ soils})$$

Stability Analysis of Finite Slopes

(i) **Fellinius Method**
(For Purely Cohesive Soil)

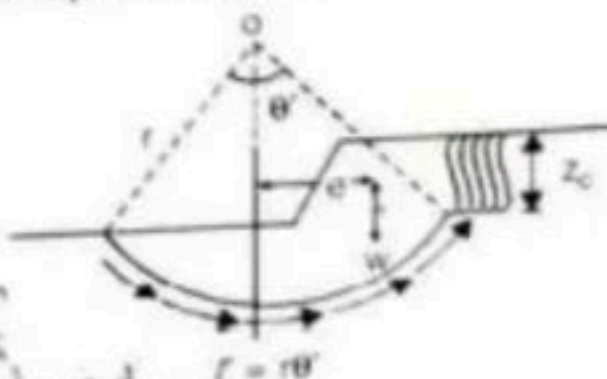
(a) $F = \frac{Cr^2\theta}{we}$

where, F = Factor of safety
 r = Radius of rupture curve
 l = Length of rupture curve



(b) $F = \frac{Cr^2\theta^1}{we}$

if tension cracks have developed. $r = 10$



F = Factor of safety if tension cracks has developed.

$$z_c = \frac{2C}{\gamma}$$

(ii) Swedish Circle Method
(Methods of slices)

$$F = \frac{Cr\theta + \sum w \cos \alpha \cdot \tan \phi}{\sum w \sin \alpha}$$

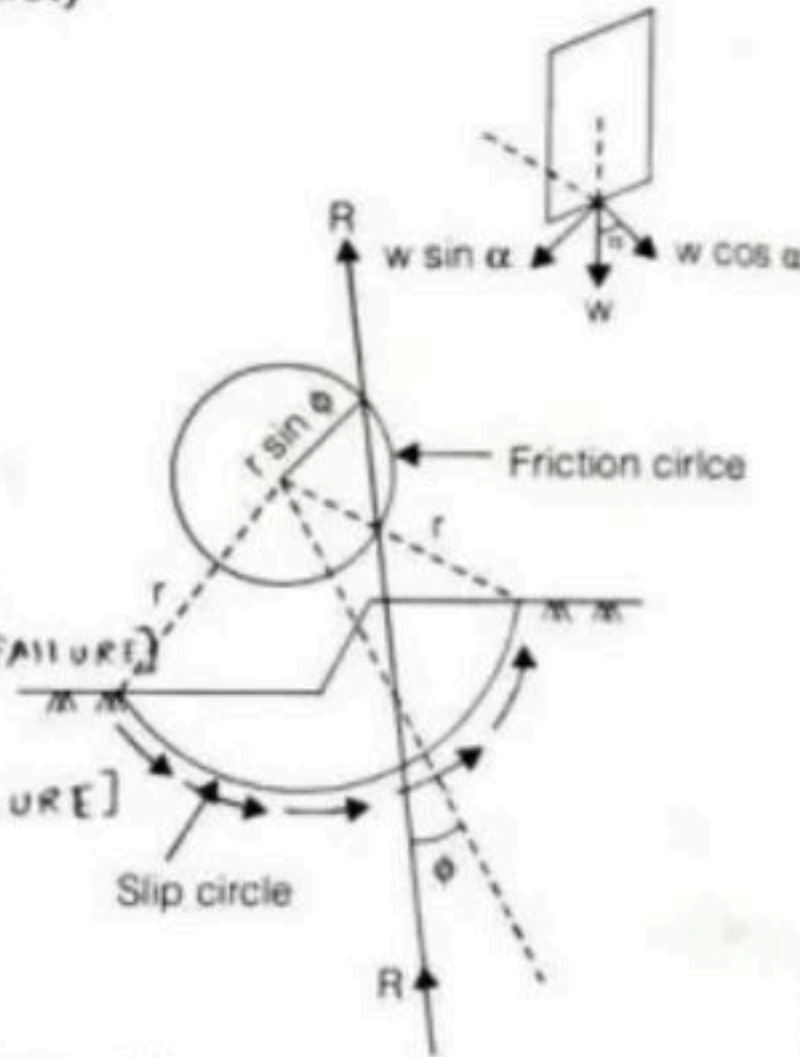
where, F = Factor of safety

(iii) Friction Circle Method

$$F_c = \frac{C}{C_m}$$

$$F_\phi = \frac{\tan \phi}{\tan \beta} = \frac{\tan \phi}{\tan \phi_m}$$

NOTE:-
For slopes $> 53^\circ$ slip circle passes through toe for any ϕ [TOE FAILURE]
For slopes $< 53^\circ$, critical circle passes below toe [BASE FAILURE]
 $\phi < 3^\circ$



(iv) Taylor's Stability Method (C-phi soil)

$$S_n = \frac{C}{\gamma H_c} = \frac{C}{\gamma F_c H}$$

In case of submerged slope γ should be used instead of γ and if slope is saturated by capillary flow they γ_{sat} should be used instead of γ .

For drawdown, from ϕ_w find S_n & new F_c .

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \cdot \phi$$

where ϕ_w = weight friction angle.

Seepage taking place

$$F.O.S. = \frac{(\gamma_{sub} \pm \cos^2 \beta) \tan \phi}{(\gamma_{sat} \pm \sin \beta \cos \beta)}$$

$$f. = \frac{\gamma_{sub} \tan \phi}{\gamma_{sat} \tan \beta} \approx \frac{1}{2} \frac{\tan \phi}{\tan \beta}$$

Seepage not taking place

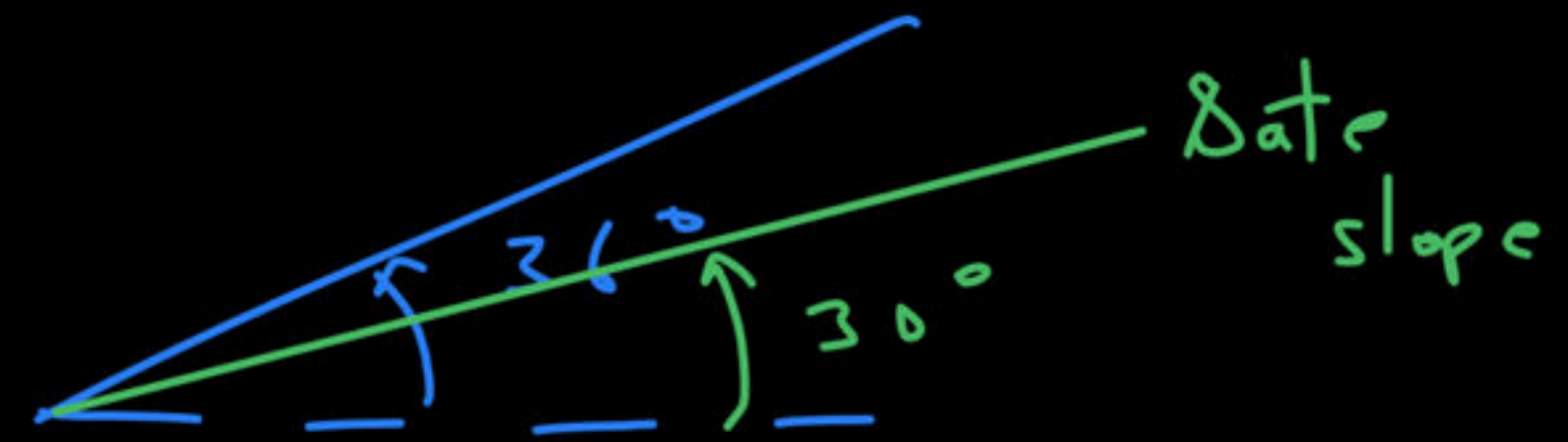
$$F. = \frac{\tan \phi}{\tan \beta}$$

A slope of infinite extent is made in a dense sand layer at angle of 30° to the horizontal. Determine the factor of safety of the slope against shear failure if the angle of internal friction of the soil be 36° .

Soln:-

$$\beta = 30^\circ$$

$$\phi = 36^\circ$$



$$F.O.S. = \frac{\tan \phi}{\tan \beta} = \frac{\tan 36^\circ}{\tan 30^\circ} = 1.26$$

∴ slope inclined at 16° to the horizontal is to be made in a cohesionless deposit having the following properties $G = 2.70$, $e = 0.72$, $\phi = 35^\circ$
 Determine the factor of safety of the slope against shear failure if water percolates in a direction parallel to the surface of the slope.

Soln:-

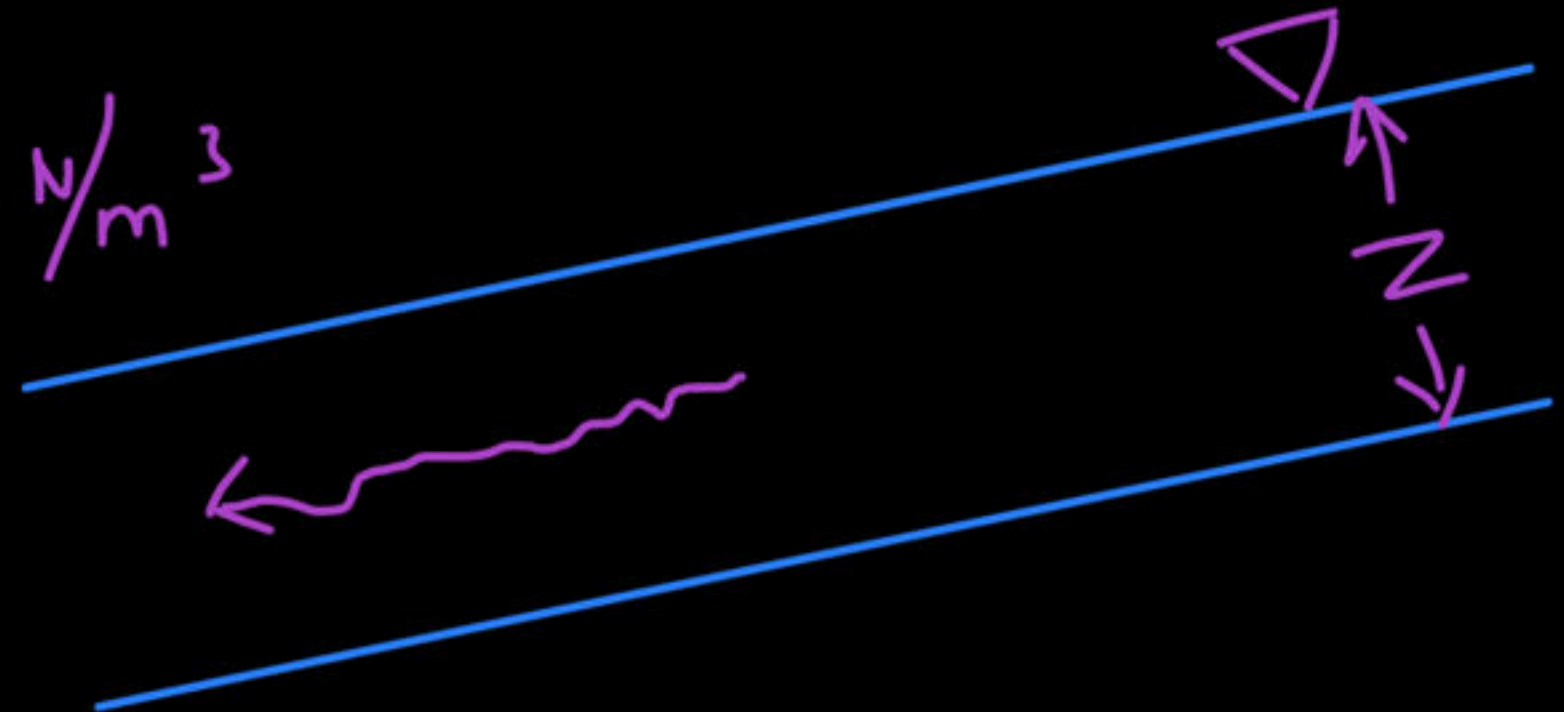
$$\gamma_{sat} = \frac{G + e}{1 + e} \gamma_w = 19.5 \text{ kN/m}^3$$

$$\gamma_{sub} = \gamma_{sat} - \gamma_w = 9.69 \text{ kN/m}^3$$

$$F.O.S. = \frac{c + \bar{\sigma} \tan \phi}{\bar{\tau}}$$

$$= \frac{0 + (\gamma_{sub} \cos \beta) \tan \phi}{(\gamma_{sat} \sin \beta)}$$

$$= \frac{9.69 \times \tan 35^\circ}{19.5 \times \tan 16^\circ} = 1.2134$$

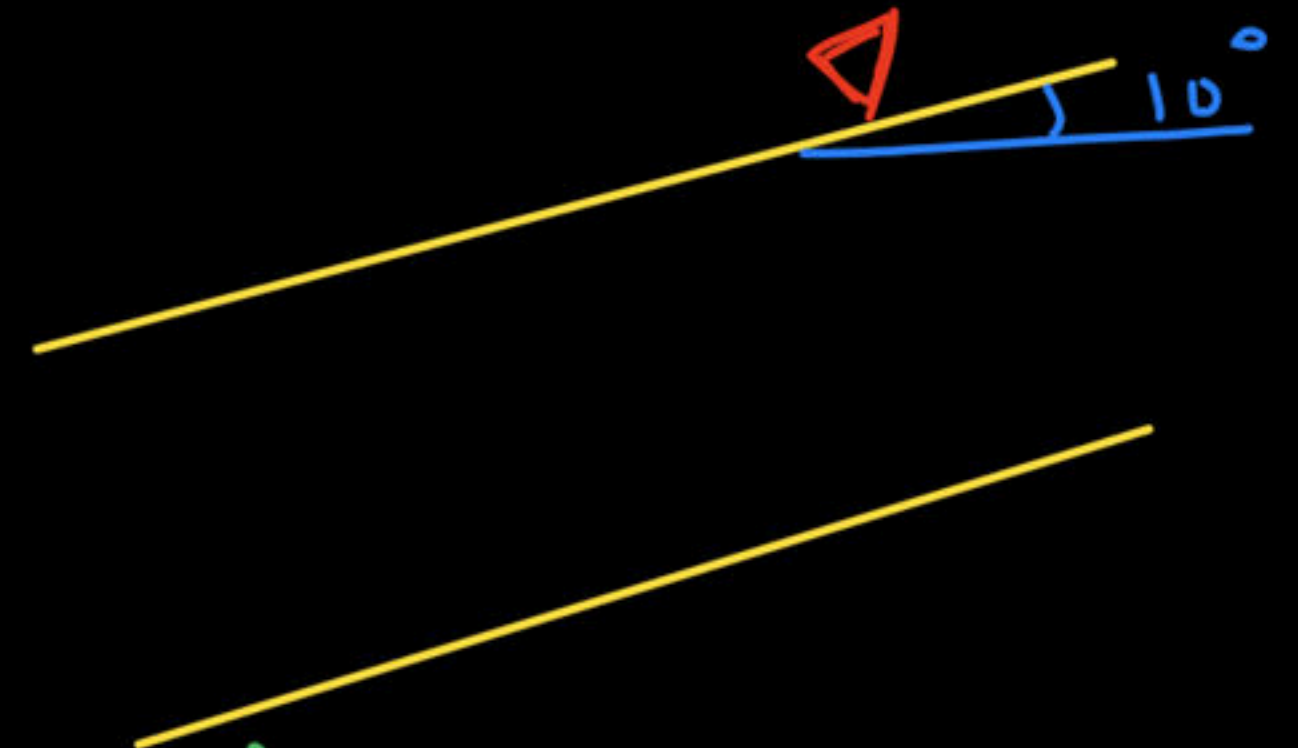


A long natural slope in an overconsolidated clay ($c' = 10 \text{ kN/m}^2$, $\phi' = 25^\circ$, $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$) is inclined at 10° to the horizontal. The water table is at the surface and seepage is parallel to the slope. If a plane slip had developed at a depth of 5 m below the surface, determine the factor of safety take $\gamma_m = 10 \text{ kN/m}^3$.

Soln:- $c' = 10 \text{ kN/m}^2$ $\phi' = 25^\circ$

$$\text{F.O.S.} = \frac{c + (\gamma_{\text{sub}} z \cos^2 \beta) \tan \phi}{(\gamma_{\text{sat}} z \sin \beta \cos \beta)}$$

$$= \frac{10 + (20 - 10) \times 5 \times \cos^2 10^\circ \tan 25^\circ}{20 \times 5 \times \sin 10^\circ \cos 10^\circ} = 1.9$$



An Embankment 10 m high is inclined at an angle of 36° to the horizontal. A stability analysis by the method of slices given the following forces per running meter

$$\Sigma \text{ Shearing forces} = 450 \text{ kN}$$

$$\Sigma \text{ Normal forces} = 900 \text{ kN}$$

$$\Sigma \text{ Neutral forces} = 216 \text{ kN}$$

The length of the failure arc is 27 m. Laboratory tests on the soil indicate the effective values c' and ϕ' as 20 kN/m^2 and 18° respectively

$$\phi = 18^\circ$$

Determine the F.O.S of the slope with respect to

- Shearing strength and
- Cohesion.



Soln:- a)
$$F.O.S. = \frac{c r A + \Sigma (N - U) \tan \phi}{\Sigma T}$$

$$F.O.S = \frac{20 \times 27 + (900 - 216) \tan 18^\circ}{450} = 1.7$$

b)
$$F.O.S. \text{ w.r.t cohesion } F_c = \frac{c r A}{\Sigma T} = \frac{20 \times 27}{450} = 1.2$$

A vertical cut is made in a clay deposit $c = 30 \text{ kN/m}^2$, $\phi = 0$, $\gamma = 16 \text{ kN/m}^3$. Find the maximum height of the cut which can be temporarily supported:

For $\phi = 0$ $S_n = 0.261$.

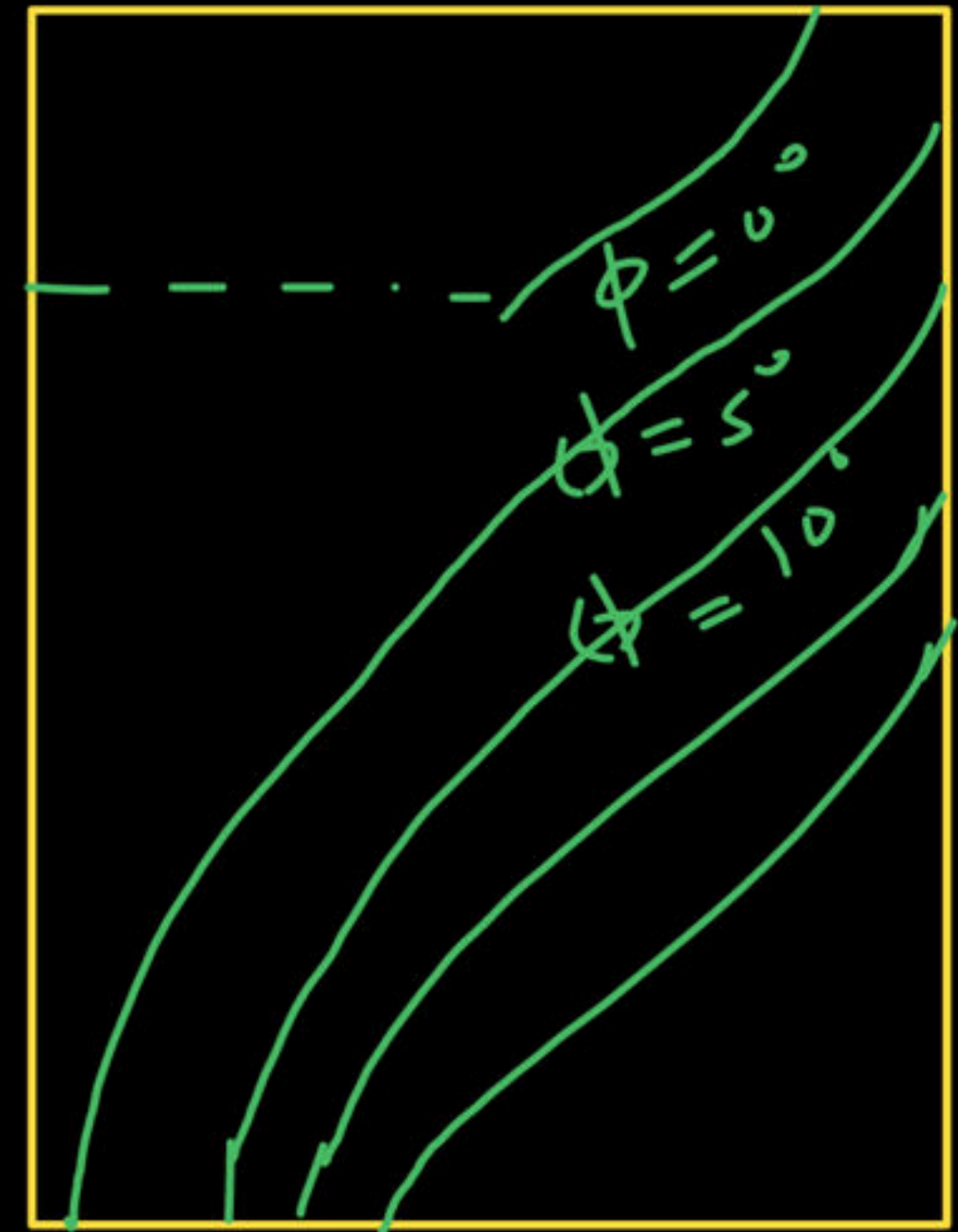
Soln:-

$$S_n = \frac{c}{\gamma H}$$

$$0.261 = \frac{30}{1 \times 16 \times H}$$

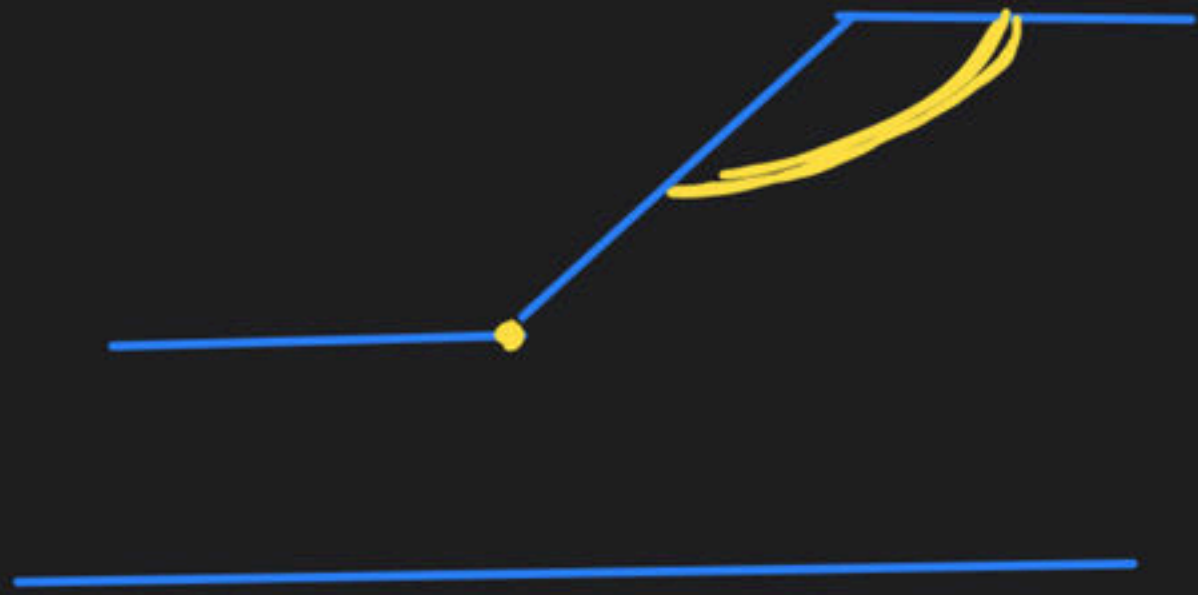
$$H_c = 7.18 \text{ m}$$

S_n



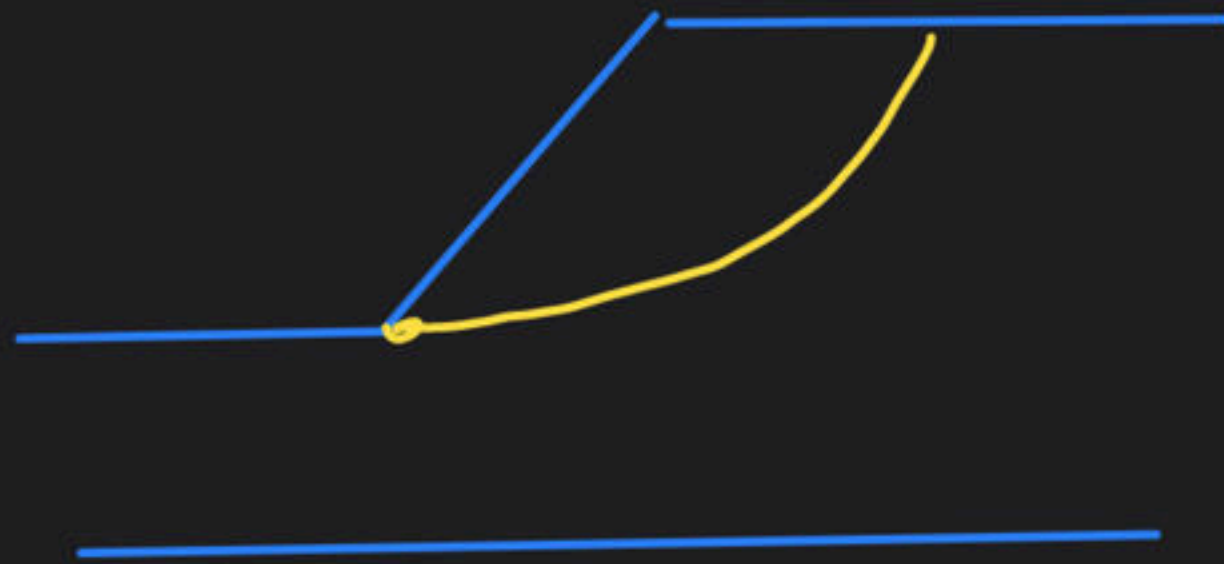
Slope angle β .

✓ For given β & ϕ , we find $S_n \rightarrow$
we find critical height.



Face / Slope failure
(soil near toe is strong)

Depth factor < 1



Toe failure
(most common)

Depth factor $= 1$



Base failure
(soil below toe is soft & slope is flat)

Depth factor > 1

An embankment is inclined at an angle of 35° and its height is 15 m. The angle of shearing resistance is 15° and cohesion intercept is 200 kN/m^2 . The unit weight of soil is 18.0 kN/m^3 . If Taylor's stability no. is 0.06 find the F.O.S with respect to cohesion.

Soln:- $\beta = 35^\circ$; $H = 15 \text{ m}$; $\phi = 15^\circ$; $C = 200 \text{ kN/m}^2$

$$S_n(\beta, \phi) = 0.06$$

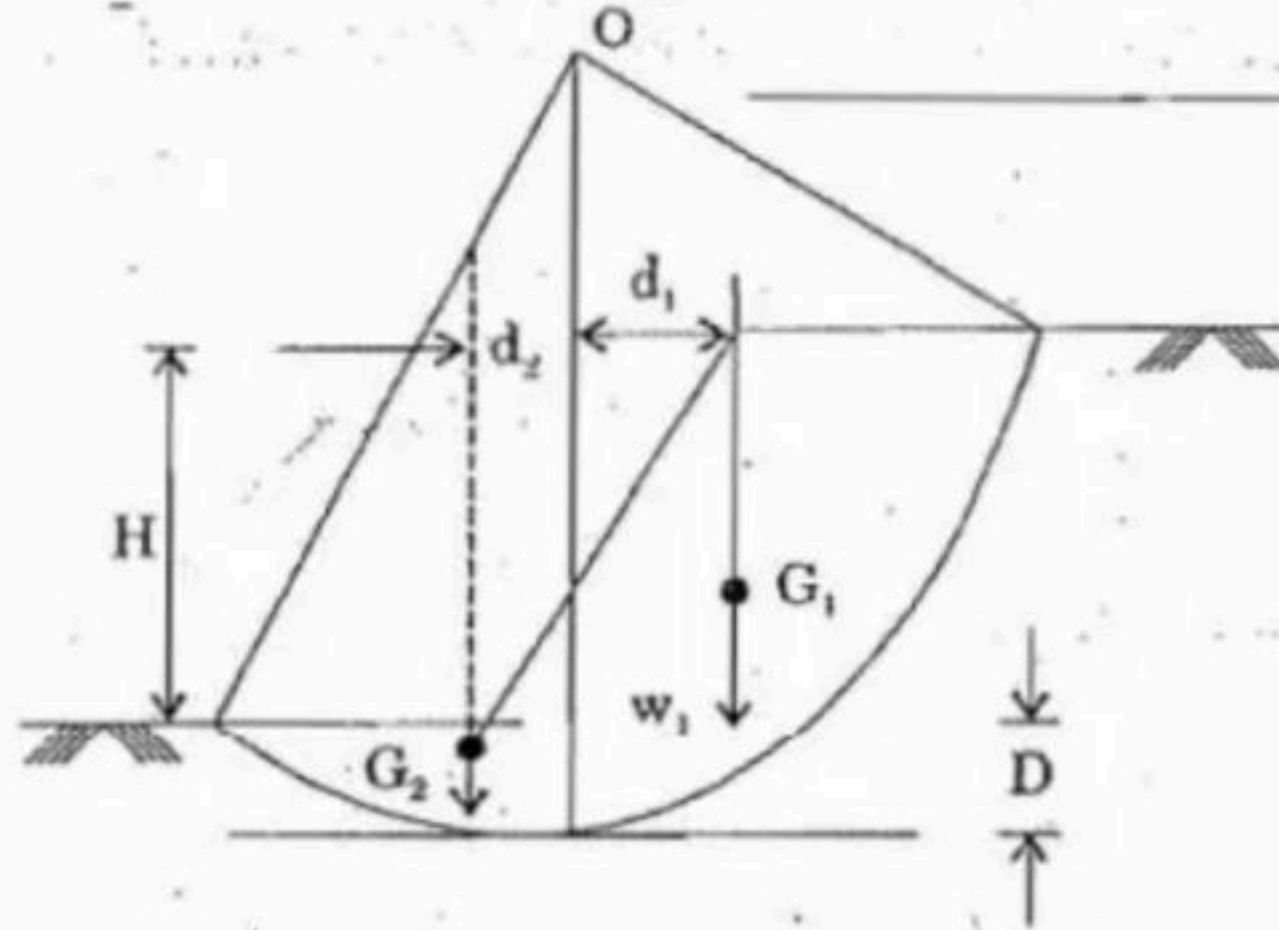
$$S_n = \frac{C}{F_c \gamma H}$$

$$0.06 = \frac{200}{F_c \times 18 \times 15}$$

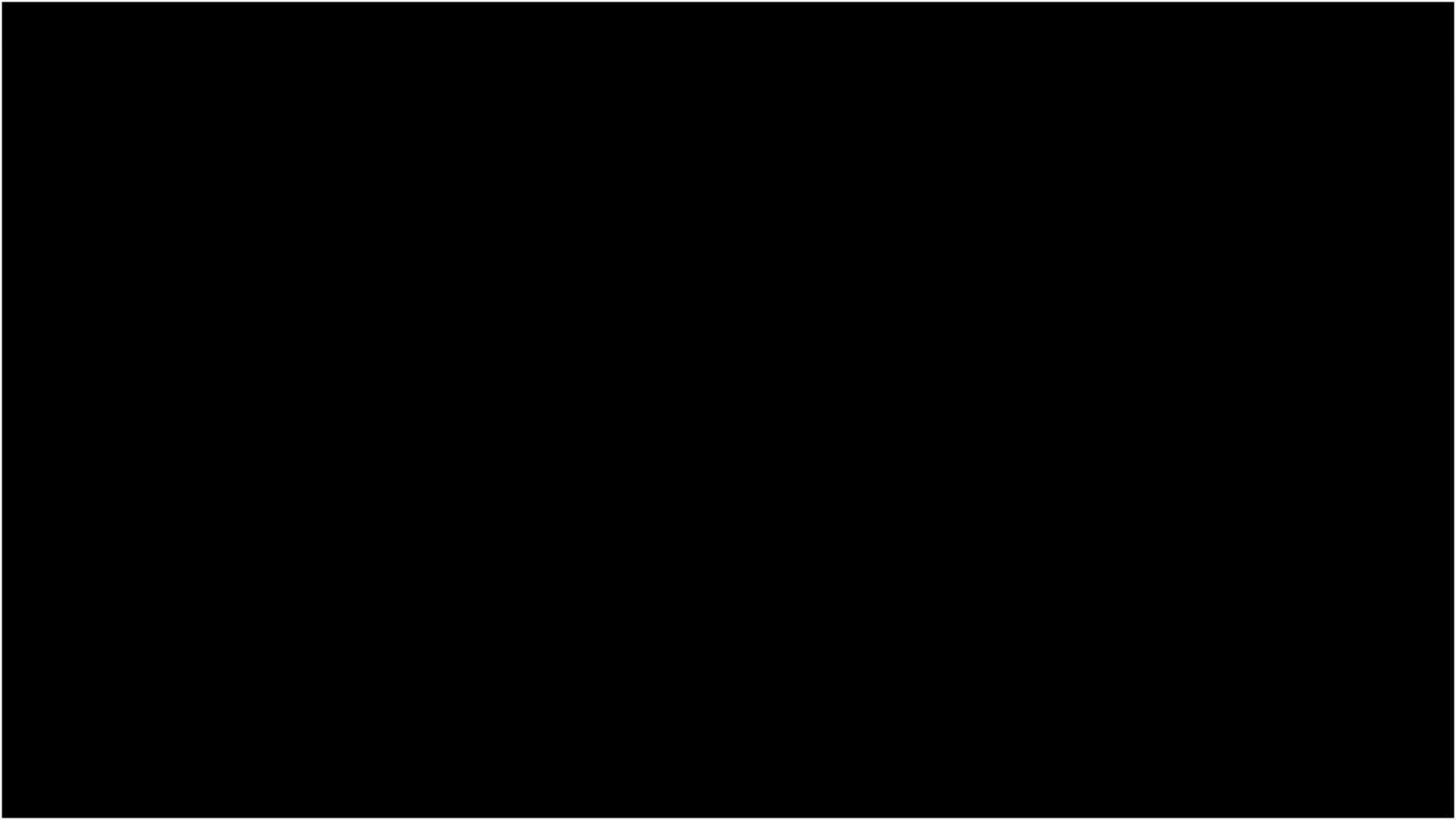
$$F_c = 12.34$$

$$\begin{aligned} F_\phi &= \frac{\tan \phi}{\tan \beta} \\ &= \frac{\tan 15^\circ}{\tan 35^\circ} \\ &= 0.38 \end{aligned}$$

The depth factor D_f in slope failure in the situation shown in the figure will be



$$\text{Depth factor } D_f = \frac{D + H}{H} > 1$$



For two infinite slopes (one in dry condition and other in submerged condition) in a sand deposit having the angle of shearing resistance 30° , factor of safety was determined as 1.5 (for both slopes). The slope angles would have been

- 21.05° for dry slope and 21.05° for submerged slope
- 19.47° for dry slope and 18.40° for submerged slope
- 18.4° for dry slope and 21.05° for submerged slope
- 22.5° for dry slope and 19.47° for submerged slope

Soln:-

Dry condition.

$$F = \frac{\tan \phi}{\tan \beta}$$

$$1.5 = \frac{\tan 30^\circ}{\tan \beta}$$

$$\beta = 21.05^\circ$$

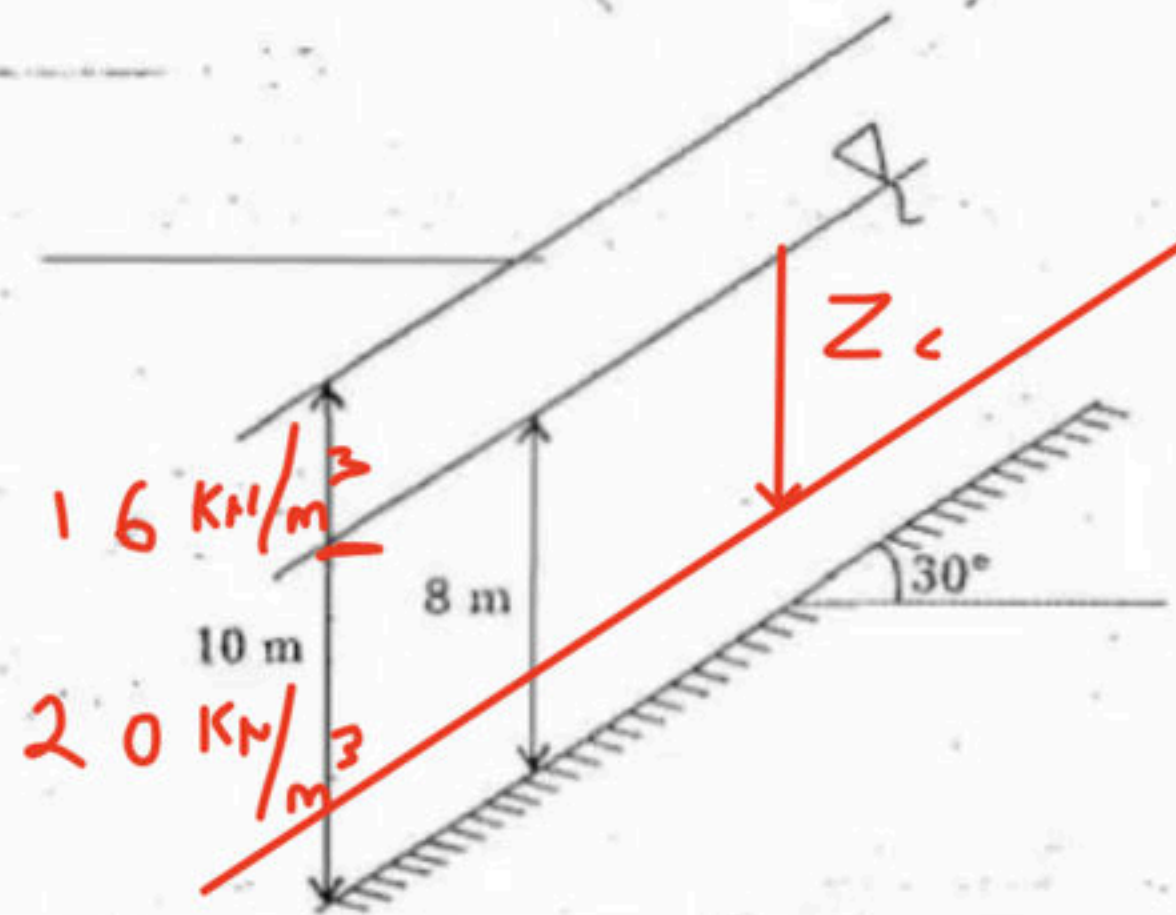
Submerged slope

$$F = \frac{\cancel{\gamma_{sub}} \cos \phi \tan \phi}{\cancel{\gamma_{sub}} \sin \beta \cancel{\cos \beta}}$$

$$1.5 = \frac{\tan \phi}{\tan \beta}$$

$$\beta = 21.05^\circ$$

The factor of safety of an infinite soil slope shown in the figure having the properties $c = 0$, $\phi = 35^\circ$, $\gamma_{dry} = 16 \text{ kN/m}^3$ and $\gamma_{sat} = 20 \text{ kN/m}^3$ is approximately equal to



Seepage takes place

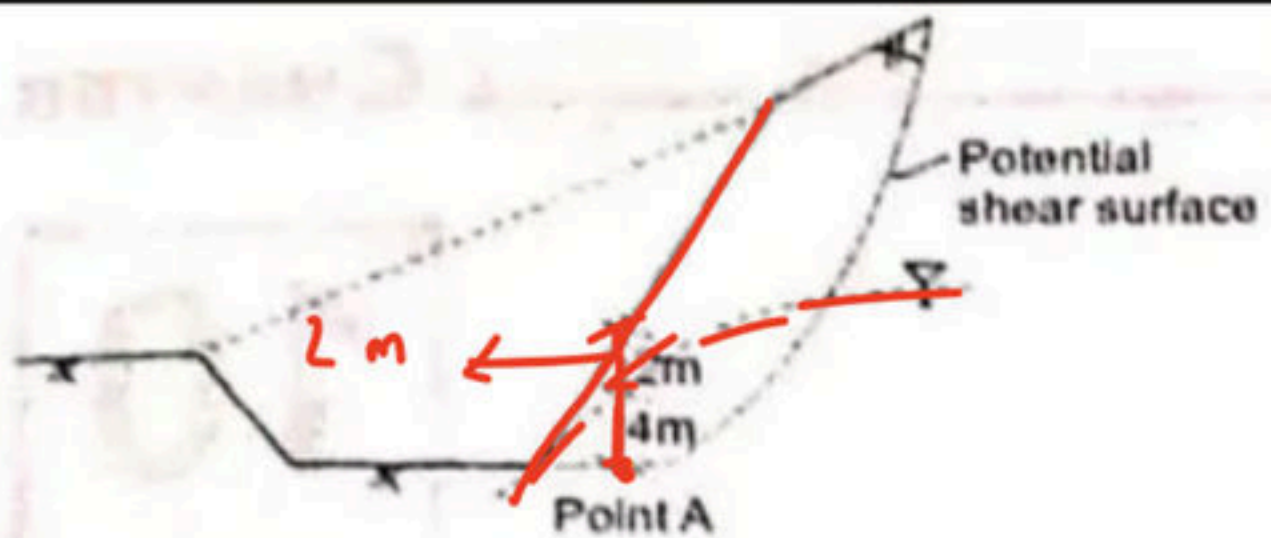
Soln:-

$$FOS = \frac{T_f}{T} = \frac{c + \bar{\sigma} \tan \phi}{\tau}$$

$$= 0 + \frac{\{16 \times 2 + (20 - 9.81) \times 8\} \tan 35^\circ}{\{16 \times 2 + 20 \times 8\} \tan 30^\circ} = 0.717$$

$$16 \times 2 + (20 - 9.81) Z_c \tan 35^\circ = (16 \times 2 + 20 \times Z_c) \tan 30^\circ$$

$$Z_c = 0.89 \text{ m}$$



The soil exhibits $c' = 20 \text{ kPa}$, $\phi' = 18^\circ$, and the undrained shear strength $= 80 \text{ kPa}$. The unit weight of water is 9.81 kN/m^3 . The unit weights of the soil above and below the ground water table are 18 and 20 kN/m^3 , respectively. If the shear stress at Point A is 50 kPa , the factors of safety against the shear failure at this point, considering the undrained and drained conditions respectively, would be

- (a) ☒ 1.6 and 0.9 (b) 0.9 and 1.6
(c) 0.6 and 1.2 (d) 1.2 and 0.6

[GATE-2017-SET-II]

$$\tau_A = 50 \text{ kPa}$$

Case I :- Undrained condition

$$F.O.S. = \frac{\text{Undrained S.S.}}{\text{Acting S.S.}} = \frac{80}{50} = 1.6$$

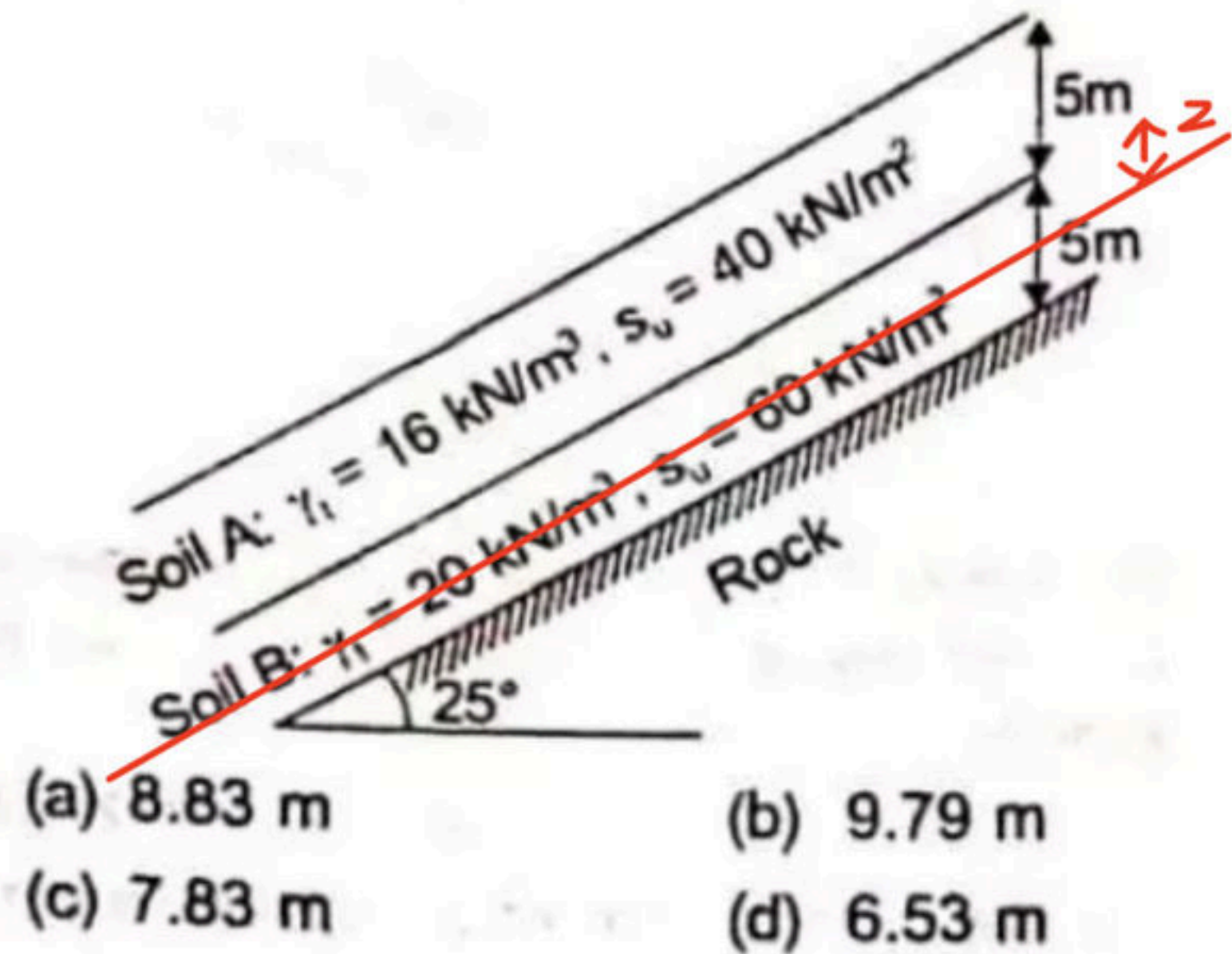
Case II :- Drained condition

$$F.O.S. = \frac{c' + \bar{\sigma} \tan \phi'}{50}$$

$$= \frac{20 + (2 \times 18 + (20 - 9.81) \times 4) \tan 18^\circ}{50}$$

$$F.O.S. = 0.9$$

The soil profile above the rock surface for a 25° infinite slope is shown in the figure, where s_u is the undrained shear strength and γ_t is total unit weight. The slip will occur at a depth d



[GATE-2013]

$$F.O.S. = \frac{c + \bar{\sigma} \tan \phi}{\tau}$$

$$\frac{40 + (16 \times 5) \cos^2 \beta \tan \phi}{16 \times 5 \cos \beta \sin \beta}$$

$$\Rightarrow 1.3 > 1$$

Hence failure plane is in Soil B.

$$c + \bar{\sigma} \tan \phi = \tau$$

$$60 + 0 = (16 \times 5 + 20z) \sin 25^\circ \cos 25^\circ$$

$$z = 3.83 \text{ m}$$

Total depth where slip occurs
8.83 m

A canal having side slopes 1 : 1 is proposed to be constructed in a cohesive soil to a depth of 10m below the ground surface. The soil properties are $\phi_u = 15^\circ$, $c_u = 12$ kPa, $e = 1.0$, $G = 2.65$.

13. If Taylor's stability number S_n is 0.08 and if the canal flows full, the factor of safety with respect to cohesion against failure of the canal bank slopes will be

- (a) 3.7 ~~(b) 1.85~~
(c) 1.0 (d) None of these

[GATE-2003]

14. If there is a sudden drawdown of water in the canal and if Taylor's stability number for the reduced value of ϕ_w is 0.126, the factor of safety with respect to cohesion against the failure of bank will be

- (a) 1.85 (b) 1.18
(c) 0.84 ~~(d) 0.53~~

$$(13) S_n = \frac{C_m}{\gamma' H}$$

$$0.08 = \frac{12}{\gamma' H}$$

$$F_c \times 8.093 \times 10$$

$$F_c = 1.85$$

$$\begin{aligned} \gamma' &= \frac{\gamma - 1}{1 + e} \times 9.81 \\ &= 8.093 \text{ kN/m}^3 \end{aligned}$$

$$(14) S_n = \frac{C}{\gamma_{sat} F_c H}$$

$$0.126 = \frac{12}{\gamma_{sat} F_c H}$$

$$\begin{aligned} &17.903 \times F_c \times 10 \\ &= 0.532 \end{aligned}$$

$$\begin{aligned} \gamma_{sat} &= \frac{G + e}{1 + e} \gamma_w \\ &= 17.903 \text{ kN/m}^3 \end{aligned}$$

An infinite slope, with a slope angle of 14° is made up of a cohesionless soil having $\phi = 30^\circ$ and $\gamma_t = 20 \text{ kN/m}^3$. It experiences seepage with the water table at the surface. If the unit weight of water is 10 kN/m^3 , the factors of safety against failure, without seepage and with seepage will be and respectively.

[GATE-1991]

$$a) \text{ F.O.S.} = \frac{\tan \phi}{\tan \beta} = \frac{\tan 30^\circ}{\tan 14^\circ} = 2.31$$

$$b) \text{ F.O.S.} = \frac{\gamma_{\text{sub}} \cdot \tan \phi}{\gamma_{\text{sat}} \tan \beta} = \left(\frac{20 - 10}{20} \right) \cdot \frac{\tan 30^\circ}{\tan 14^\circ} = 1.16$$

(seepage takes place)